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Supra i-Open Sets and Supra i-Continuity on Topological Spaces

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Abstract—In this paper, we introduce and investigate a new class of sets and maps between topological spaces called supra i-open sets and supra i-continuous maps, respectively. Furthermore, we introduce the concepts of supra i-open maps and supra i-closed maps and investigate several properties of them.

Index Terms—supra open set, supra continuity,supra i-open set, supra i-continuity, supra i-open map, supra i-closed map and supra topological space.

1 Introduction

In 1963, Levine N. [2] introduced the concept of semi-open sets. In 1965, Njastad O. [5] introduced the concept of α-open sets. In 1983, Mashhour A. S. et al. [3] introduced the supra topological spaces and studied s-continuous maps and $s*$-continuous maps. In 2008, Devi R. et al. [1] introduced and studied a class of sets and maps between topological spaces called supra α-open sets and supra α-continuous maps, respectively. In 2012, Mohammed A. A. and Askandar S. W. [4] introduced the concept of i-open sets which they could to entire them together with many other concepts of Generalized open sets Now, we introduce the concept of supra i-open sets and study some basic properties of it. Also, we introduce the concepts of supra i-continuous maps, supra i-open maps and supra i-closed maps and investigate several properties for these classes of maps. In particular, we study the relation between supra i-continuous maps and supra i-open maps (supra i-closed maps). Throughout this paper, (X,τ), (Y,σ) and (Z,η) (or simply, X , Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. All sets are assumed to be subset of topological spaces. The closure and the interior of a set A are denoted by $cl(A)$and $int(A)$, respectively. The complement of the subset A of X is denoted by $X\backslash A$. A subcollection $\psi \subseteq 2^X$ is called a supra topology [3] on X if $X \in \psi$ and ψ is closed under arbitrary union. $(X,\tau)$is called a supra topological space. The elements of ψ are called supra open in (X,τ) and the complement of a supra open set is called a supra closed set. The supra closure of a set A, denoted by $cl(A)$, is the intersection of the supra closed sets including A. The supra interior of a set$A$, denoted by $int(A)$, is the union of the supra open sets included in A.The supra topology ψ on X is associated with the topology τ if $\tau \subseteq \psi$. A set A is called supra semi-open [6](resp. supra α-open [1]) if $A \subseteq cl\psi(int\psi(A))$ (resp. $A \subseteq int\psi(cl\psi(int\psi(A)))$).

2 Supra *i*-open sets

In this section, we introduce a new class of open sets called supra i-open set and study some of their basic properties.

Definition 2.1: Let (X,τ) be a supra topological space. A set A is called supra i-open set if$A \subseteq cl(A \cap O^{\psi})$ where $O^{\psi} \in \psi$and $\psi \neq X,\phi$. The complement of supra i-open set is called supra i-closed set.

Example 2.1: Let (X,ψ) be a supra topological space, where $X=\{a,b,c,d\}$and $\psi=\{\phi,\{a\},\{b\},\{a,b\},X\}$. Here $\{b,c\}$ is a supra i-open set.

Remark 2.1:

(i) The union of supra i-open sets may fail to be a supra i-open set.
(ii) The intersection of supra i-open sets may fail to be a supra i-open set.

Theorem 2.1: Every supra semi-open set is supra i-open.

Proof: let A be a supra semi-open set in supra topological space (X,τ). By definition of supra semi-open sets there exists a supra open set S^O such that $S^O \subseteq A \subseteq cl_{\psi}(S^O)$. Since $S^O \subseteq A$then $A \cap S^O = S^O$. Hence $A \subseteq cl(A \cap S^O)$. This is means that A is supra i-open.

The converse of the above theorem need not be true. This is shown by the following example.

Example 2.2: Let (X,ψ) be a supra topological space, where $X=\{a,b,c\}$and $\psi=\{\phi,\{a\},\{a,b\},\{b,c\},X\}$. Here $\{c\}$ is a supra i-open set, but it is not supra semi-open.

Corollary 2.1: Every supra α-open set is supra i-open.

Proof: by theorem 3.2 in [1] and theorem 2.1.

Corollary 2.2: Every supra open set is supra i-open.

Proof: by theorem 3.1 in [1] and corollary 2.1.

Definition 2.2: The supra i-closure of a set A, denoted by $cl_i^{\psi}(A)$is the intersection of supra i-closed sets including A. The supra i-interior of a set A, denoted by $int_i^{\psi}(A)$, is the union of supra i-open sets included in A.

Remark 2.2: It is clear that $int_i^{\psi}(A)$ is a supra i-open set and $cl_i^{\psi}(A)$ is a supra i-closed set.

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Theorem 2.2:

(i) $A \subseteq cl_i^{\psi}(A)$; and A=$cl_i^{\psi}(A)$ if and only if A is a supra i-closed set;
(ii) $int_i^{\psi}(A)$⊆A; and A=$int_i^{\psi}(A)$ if and only if A is a supra i-open set;
(iii)$X \setminus int_i^{\psi}(A)$=$cl_i^{\psi}(A)$;
(iv)$X \setminus cl_i^{\psi}(A)$=$int_i^{\psi}(A)$.

Proof: obvious.

Theorem 2.3:

(i) $int_i^{\psi}(A) \cup int_i^{\psi}(B) \subseteq int_i^{\psi}(A \cup B)$;
(ii) $cl_i^{\psi}(A \cap B) \subseteq cl_i^{\psi}(A) \cap cl_i^{\psi}(B)$.

Proof: obvious.

3 SUPRA *i*-CONTINUOUS MAPS

In this section, we introduce a new type of continuous maps called a supra i-continuous map and obtain some of their properties and characterizations.

Definition 3.1: Let (X,τ) and (Y,σ) be two topological spaces and ψ be an associated supra topology with τ. A map $f:(X,\tau) \to (Y,\sigma)$ is called a supra i-continuous map if the inverse image of each open set in Y is a supra i-open set in X.

Theorem 3.1: Let (X,τ) and (Y,σ) be two topological spaces and ψ be an associated supra topology with τ. Let f be a map from Xinto Y. Then the following are equivalent.

(1) f is a supra i-continuousmap;
(2) The inverse image of a closed set in Y is a supra i-closed set in X;
(3) $cl_i^{\psi}(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every A in Y;
(4) $f(cl_i^{\psi}(A)) \subseteq cl(f(A))$ for every A in X;
(5) $f^{-1}(int(B)) \subseteq int_i^{\psi}(f^{-1}(B))$for every B in Y.

Proof:(1)⇒(2): Let A be a closed set in Y, then $Y \setminus A$ is an open set in Y . Then $f^{-1}(Y \setminus A) = X \setminus f^{-1}(A)$ is a supra i-open set in X. It follows that $f^{-1}(A)$ is a supra i-closed subset of X.

(2)⇒(3):Let A be any subset of Y. Since $cl(A)$ is closed in Y, then $f^{-1}(cl(A))$ is supra i-closed in X. Therefore, $cl_i^{\psi}(f^{-1}(A)) \subseteq cl_i^{\psi}(f^{-1}(cl(A))) \subseteq f^{-1}(cl(A))$.

(3)⇒(4): Let A be any subset of X. By (3) we have $cl_i^{\psi}(A) \subseteq cl_i^{\psi}(f^{-1}(f(A))) \subseteq f^{-1}(cl(f(A)))$.Therefore, we have $f(cl_i^{\psi}(A)) \subseteq cl(f(A))$.

(4)⇒(5): Let B be any subset of Y. By (4), $f(cl_i^{\psi}(X \setminus f^{-1}(B))) \subseteq cl(f(X \setminus f^{-1}(B)))$and $f(X \setminus int_i^{\psi}(f^{-1}(B))) \subseteq cl(Y \setminus B) = Y \setminus B$. Therefore, we have $X \setminus int_i^{\psi}(f^{-1}(B)) \subseteq f^{-1}(Y \setminus int(B))$ and $f^{-1}(int(B)) \subseteq int_i^{\psi}(f^{-1}(B))$.

(5)⇒(1): Let B be an open set in Y and $f^{-1}(int(B)) \subseteq int_i^{\psi}(f^{-1}(B))$.Then, $f^{-1}(B) \subseteq int_i^{\psi}(f^{-1}(B))$. But, $int_i^{\psi}(f^{-1}(B)) \subseteq f-1(B)$. Hence, $f^{-1}(B) = int_i^{\psi}(f^{-1}(B))$. Therefore, $f^{-1}(B)$ is supra i-open in X.

Theorem 3.2: Let (X,τ) and (Y,σ) be two topological spaces and ψ and ζ be the associated supra topologies with $\tau$and σ, respectively. Then $f:(X,\tau) \to (Y,\sigma)$ is a supra i-continuous map, if one of the following holds:

(1) $f^{-1}(int_i^{\zeta}(B)) \subseteq int(f^{-1}(B))$for every set B in Y.
(2) $cl(f^{-1}(B)) \subseteq f^{-1}(cl_i^{\zeta}(B))$ for every set B in Y.
(3) $f(cl(A)) \subseteq cl_i^{\psi}(f(A))$for every set A in X.

Proof: Let B be any open set of . If condition (1) is satisfied, then$f^{-1}(int_i^{\zeta}(B)) \subseteq int(f^{-1}(B))$. We get $f^{-1}(B) \subseteq int(f^{-1}(B))$. Therefore, $f^{-1}(B)$ is an open set.Every open set is supra i-open. Hence, f is a supra i-continuous map.

If condition (2) is satisfied, then we can easily prove that f is a supra i-continuous map.

Let condition (3) be satisfied and B be any open set of Y. Then $f^{-1}(B)$ is a set in $X$and $f(cl(f^{-1}(B))) \subseteq cl_i^{\psi}(f(f^{-1}(B)))$. This implies $(cl(f^{-1}(B))) \subseteq cl_i^{\psi}(B)$. This is nothing but condition (2). Hence f is a supra i-continuous map.

4SUPRA *i*-OPEN MAPS AND SUPRA *i*-CLOSED MAPS

Definition4.1: A map $f:(X,\tau) \to (Y,\sigma)$ is called a supra i-open (resp. supra i-closed) if the image of each open (resp. closed) set in X is supra i-open (resp. supra i-closed) in Y.

Theorem 4.1: A map $f:(X,\tau) \to (Y,\sigma)$ is supra i-open if and only if $f(int(A)) \subseteq int_i^{\psi}(f(A))$ for each set A in X.

Proof: Suppose that f is a supra i-open map. Since $int(A) \subseteq A$, then $f(int(A)) \subseteq f(A)$. By hypothesis, $f(int(A))$ is a supra i-open set and $int_i^{\psi}(f(A))$ is the largest supra i-open set contained in $f(A)$. Hence $f(int(A)) \subseteq int_i^{\psi}(f(A))$.**Conversely,** suppose A is an open set in X. Then $int(A) = A$, since $f(int(A)) \subseteq int_i^{\psi}(f(A))$, then $f(A) \subseteq int_i^{\psi}(f(A))$. Therefore $f(A)$ is a supra i-open set in Y and f is a supra i-open map.

Theorem 4.2: A map $f:(X,\tau) \to (Y,\sigma)$ is supra i-closed if and only if $cl_i^{\psi}(f(A)) \subseteq f(cl(A))$for each set Ain X.

Proof: Suppose f is a supra i-closed map. Since for each set Ain X, $c(A)$ is closed set in X, then $f(cl(A))$ is a supra i-closed set in Y . Also, since $f(A) \subseteq f(cl(A))$, then $cl_i^{\psi}(f(A)) \subseteq f(cl(A))$.

Conversely, Let A be a closed set in X. Then $A = cl(A)$, and Since $cl_i^{\psi}(f(A))$ is the smallest supra i-closed set containing $f(A)$, then $f(A) \subseteq cl_i^{\psi}(f(A)) \subseteq f(cl(A)) = f(A)$. Thus, $cl_i(f(A)) = f(A)$. Hence, $f(A)$ is a supra i-closed set in Y. Therefore, f is a supra i-closed map.

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Theorem 4.3: Let $(X,\tau),(Y,\sigma)$ and (Z,η) be three topological spaces and $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$be two maps. Then,

(1) if$g\circ f$ is supra i-open and f is continuous surjective, then g is a supra$i$-open map.
(2) if$g\circ f$ is open and g is supra i-continuous injective, then f is a supra$i$-open map.

Proof:

(1) Let A be an open set in . Then,$f^{-1}(A)$ is an open set in X. Since $g\circ f$ is a supra i-open map, then $(g\circ f)(f^{-1}(A)) = g(f(f^{-1}(A))) = g(A)$ (because f is surjective) is a supra i-open set in Z. Therefore, g is a supra i-open map.
(2) Let A be an open set in X. Then, $g(f(A))$ is an open set in Z. Therefore, $g^{-1}(g(f(A))) = f(A)$ (because g is injective) is a supra i-open set in Y. Hence, f is a supra i-open map.

Theorem 4.4: Let (X,τ) and (Y,σ) be two topological spaces and $f:(X,\tau)\to(Y,\sigma)$ be a bijective map. Then the following are equivalent:

(1) f is a supra i-open map;
(2) f is a supra i-closed map;
(3) $f^{-1}$is a supra$i$-continuous map.

Proof: (1)⇒(2): Suppose B is a closed set in X. Then $X\backslash B$ is an open set in X and by (1), $f(X\backslash B)$ is a supra i-open set in Y. Since f is bijective, then $f(X\backslash B) = Y\backslash f(B)$.Hence, $f(B)$ is a supra i-closed set in Y. Therefore, f is a supra i-closed map.

(2)⇒(3): Let f is a supra i-closed map and B be closed set in X. Since f is bijective, then $(f^{-1})^{-1}(B) = f(B)$ which is a supra i-closed set in Y. Therefore, by Theorem 3.1, f is a supra i-continuous map.

(3)⇒(1): Let A be an open set in X. Since f^{-1} is a supra i-continuous map, then $(f^{-1})^{-1}(A) = f(A)$ is a supra i-open set in Y. Hence, f is a supra i-open map.

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